Ratios, Rates, and Conversions

* ratio: Compares two numbers by division.
  Written as a/b $a:b$ and a to b
* rate: a ratio that compares quantities measured in different units.
* Unit rate: a rate with a denominator of 1 unit
* Conversion rate: is a ratio of two equivalent measures in different units.

$\frac{4}{5}$ 3:8 40 to 10 ← Are ratios but not unit ratios because the denominator is not 1.

$\frac{11.25}{1}$ 375:1 4 to 1 ← these are unit ratios because the denominator is 1.

* Any ratio can be converted into a unit ratio by dividing the numerator and the denominator by the denominator.

Algebraic:

$$\frac{a}{b} = \frac{a \div b}{b \div b} = \frac{a \div b}{1}$$

Numeric Example:

$$\frac{\$25}{2} = \frac{\$25 \div 2}{2 \div 2} = \frac{\$12.50}{1}$$

Example: Trisha ran 10km in 2.5h. Jason ran 7.5km in 2h. Olga ran 9.5km in 2.25h. Who had the fastest average speed?

Trisha: $\frac{10\text{km} \div 2.5}{2.5\text{h} \div 2.5} = \frac{4\text{km}}{1\text{hr.}}$

Jason: $\frac{7.5\text{km} \div 2}{2\text{h} \div 2} = \frac{3.75\text{km}}{1\text{h}}$

Olga: $\frac{9.5\text{km} \div 2.25}{2.25\text{h} \div 2.25} = \frac{4.\bar{2}\text{ km}}{1\text{ hr.}}$

↑ covered the most distance in 1 hr, so she is going the fastest.

Converting Units

Conversion factor: a ratio of two equivalent measures in different units.

* Choose and multiply by the conversion factor.
* The appropriate factor divides out the common units.

Example1: 330min; hours

$330\text{min} \cdot \frac{1\text{h}}{60\text{min}}$ ← equivalent measures ← choose conversion factor

$= 330\cancel{\text{min}} \cdot \frac{1\text{h}}{60\cancel{\text{min}}}$ ← Divide out common units

$= 5.5$ hours ← simplify

Converting Units

Example 2: 15 kg; grams

$15\text{kg} \cdot \frac{1000\text{g}}{1\text{kg}}$ ← equivalent measures

$15\text{kg} \cdot \frac{1000}{1\text{kg}}$

$= 15 \cdot 1000\text{g} = \boxed{15{,}000\text{g}}$

Example 3: 5 ft 3 in; inches

$5\text{ft}\ 3\text{in} = 5\text{ft} + 3\text{in}$

$= 5\cancel{\text{ft}} \cdot \frac{12\text{in}}{1\cancel{\text{ft}}} + 3\text{in}$

$= 5 \cdot 12\text{in} + 3\text{in}$

$= 60\text{in} + 3\text{in}$

$= \boxed{63\text{in}}$

The CN Tower in Toronto, Canada, is about 1815 ft. tall. About how many meters tall is the tower? Use the fact that $1\text{m} \approx 3.28\text{ft}$.

$1815\text{ft} \cdot \frac{1\text{m}}{3.28\text{ft}}$

$1815\cancel{\text{ft}} \cdot \frac{1\text{m}}{3.28\cancel{\text{ft}}}$

$\frac{1815 \cdot 1\text{m}}{3.28} \approx \boxed{553\text{ meters}}$

Converting Units

Monetary exchange rates change from day to day. On one day the exchange rate for dollars to euros was about 1 dollar = .63 euro. How many euros could you get for $325?

$$\$325 \cdot \frac{.63 \text{ euro}}{\$1} = 325 \cdot 0.63 = \boxed{204.75 \text{ euros}}$$

An athlete ran a sprint of 100 ft in 3.1 s. At what speed was the athlete running in miles per hour? Round to the nearest mile per hour.

$$\frac{100\text{ft}}{3.1\text{s}} \cdot \frac{1\text{mi}}{5280\text{ft}} \cdot \frac{60\text{s}}{1\text{min}} \cdot \frac{60\text{min}}{1\text{hr}} = \frac{100 \cdot 1\text{mi} \cdot 60 \cdot 60}{3.1 \cdot 5280 \cdot 1 \cdot 1\text{hr}} = \frac{360000}{16368}$$

$$= \boxed{\frac{22\text{m}}{1\text{hr}}}$$

The janitor at a school discovered a slow leak in a pipe. The janitor found that it was leaking at a rate of 4 fl. oz. per minute. How fast was the pipe leaking in gallons per hour?

$$\frac{4\text{oz}}{1\text{min}} \cdot \frac{1\text{cup}}{8\text{oz}} \cdot \frac{1\text{ gal.}}{16\text{cup}} \cdot \frac{60\text{min}}{1\text{ hr.}} = \frac{4 \cdot 1 \cdot 1\text{gal} \cdot 60}{1 \cdot 8 \cdot 16 \cdot 1\text{hr}} = \frac{240\text{gal}}{128\text{ hr.}}$$

$$\frac{240\text{gal} \div 128}{128\text{ hr} \div 128} = \boxed{\frac{1.875\text{ gal}}{1\text{hr}}}$$